# Chromaticity Measurements using Phase Modulated RF and Vector Signal Analyzers


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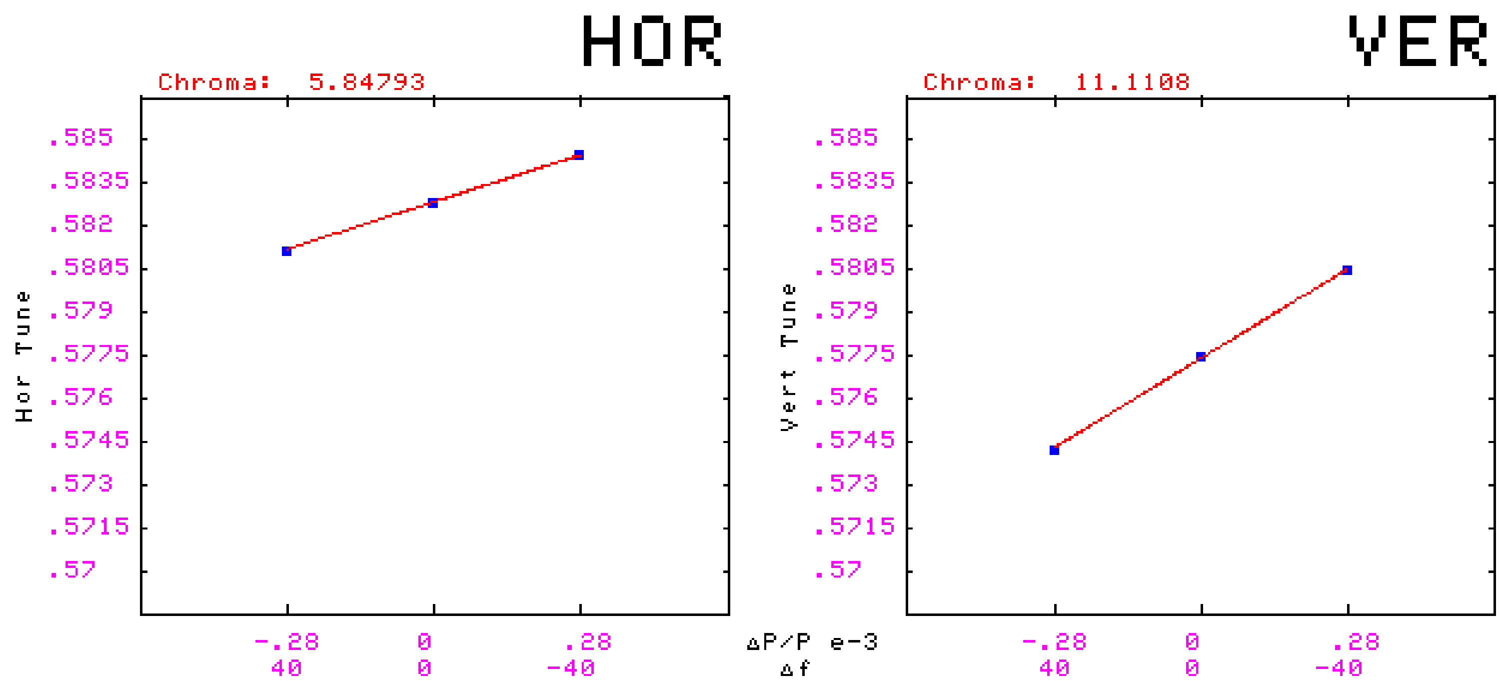
See: http://www-bdnew.fnal.gov/pbar/documents/pbarnotes/pdf_files/PB656.PDF

# Chromaticity and Tune Measurements in the TEVATRON

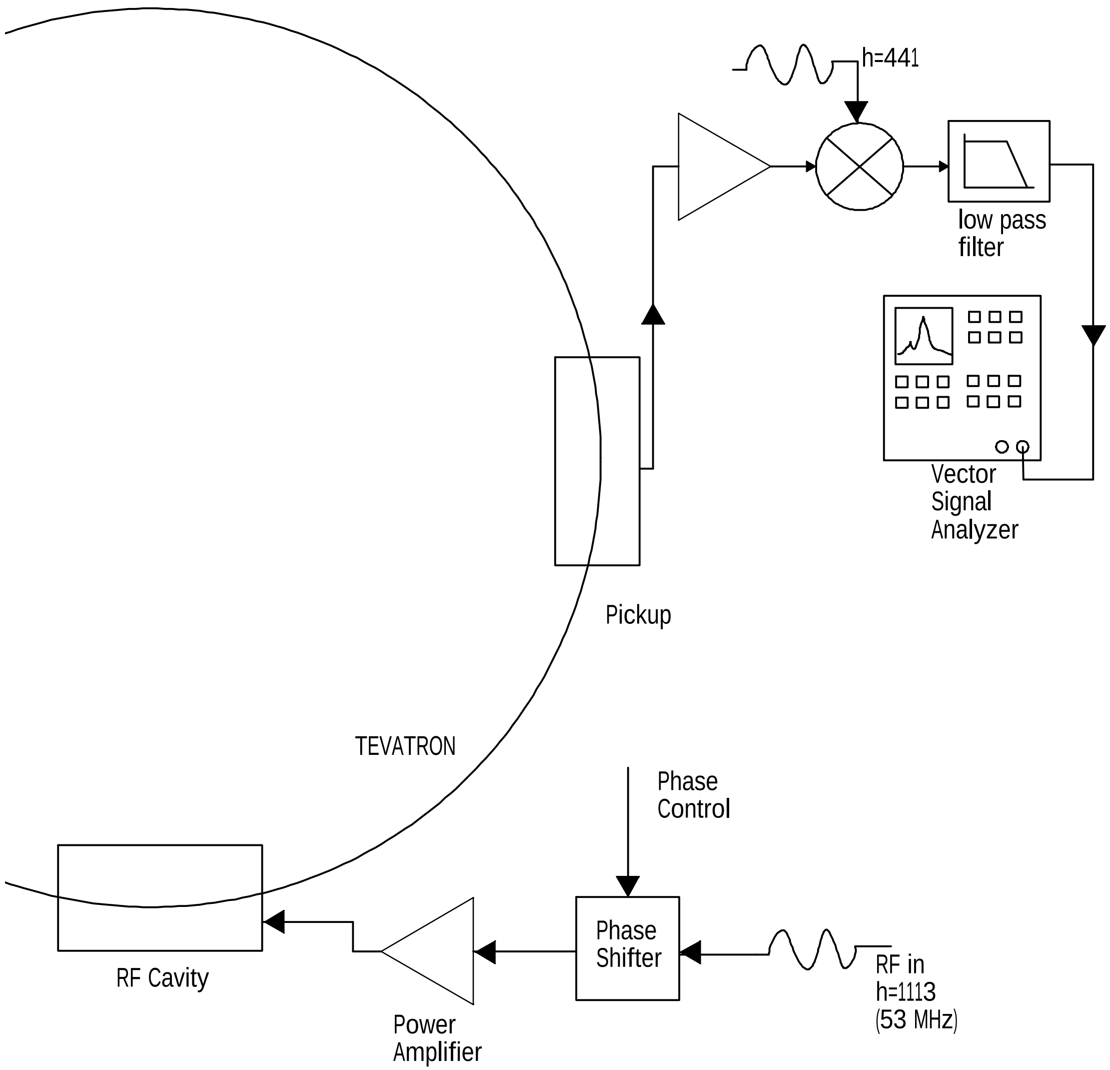
- The betatron motion is excited by the “residual noise” of the TEVATRON (power supply ripple, kickers, vibrations, RF noise, etc.)
- Betatron signal is measured with stripline pickups resonant at 21 MHz (harmonic 441).
  - The signal is down converted to baseband with a sine wave at harmonic 441
  - The signal is analyzed with vector signal analyzers (FFT Box)
- The TEVATRON operates with a rather large chromaticity (>6) to combat head-tail instabilities.
  - The TEVATRON does not have transverse dampers in collider mode because of the emittance growth due to the noise of the dampers
  - Because of the large chromaticity, there are typically 5-10 synchrotron lines modulating the betatron lines
    - Revolution Frequency = 47 kHz
    - Betatron Frequency = 20 kHz
    - Synchrotron Frequency = 80 Hz (at injection)

# Chromaticity and Tune Measurements in the TEVATRON

- To measure the chromaticity, the beam energy is changed by changing the RF frequency.
  - The beam energy is changed by about +/- 0.03% so that an appreciable shift in betatron tune is observed.
  - The measurement of betatron tune is complicated by the large amount of synchrotron lines.



# RF Phase Modulation Method for Measuring the Chromaticity in the TEVATRON




- Fermilab is currently investigating a RF phase modulation technique for measuring the chromaticity in the TEVATRON
- In this method, the beam energy is modulated at a very low frequency (<< synchrotron frequency) by phase shifting the RF drive to the cavities.
- The signal is analyzed by a vector signal analyzer that uses digital PM demodulation algorithms.

# Betatron Motion with Phase Modulated RF

- RF Phase modulation

$$\phi_{rf} = \omega_{rf} t + \Delta\phi_{rf} \sin(\Omega_{mod} t)$$

- Energy modulation

Synchrotron motion

$$\frac{\Delta pc(t)}{pc} = -\frac{1}{\eta\omega_r}\left(\Omega_{mod}\frac{\Delta\phi_{rf}}{h}\cos(\Omega_{mod} t) + \Omega_s \Delta\phi_s \cos(\Omega_s t + \theta_s)\right)$$

Synchrotron amplitude

Synchrotron frequency

- Betatron frequency at the k$^{th}$ harmonic

$$\frac{d\phi_{k_\pm}}{dt} = (k \pm Q_o)\omega_r + \left(k \pm Q_o \mp \frac{\chi}{\eta}\right)\frac{\Omega_{mod}\Delta\phi_{rf}}{h}\cos(\Omega_{mod} t) + \left(k \pm Q_o \mp \frac{\chi}{\eta}\right)\Omega_s \Delta\phi_s \cos(\Omega_s t)$$

where $Q_o$ is the fractional tune and $\chi$ is the chromaticity

# Beam Current Spectrum with Phase Modulated RF

$$I_\Delta = \frac{\omega_r q_b}{2} \frac{A}{\sqrt{\varepsilon_{max}}} \sum_{k=0}^{\infty} C_k \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} J_m(Y_+) J_n(Z_+) \cos\left(\omega_{k,m,n}^{+} t + \psi_+\right)$$

$$+ \frac{\omega_r q_b}{2} \frac{A}{\sqrt{\varepsilon_{max}}} \sum_{k=0}^{\infty} C_k \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} J_m(Y_-) J_n(Z_-) \cos\left(\omega_{k,m,n}^{-} t + \psi_-\right)$$

Betatron amplitude

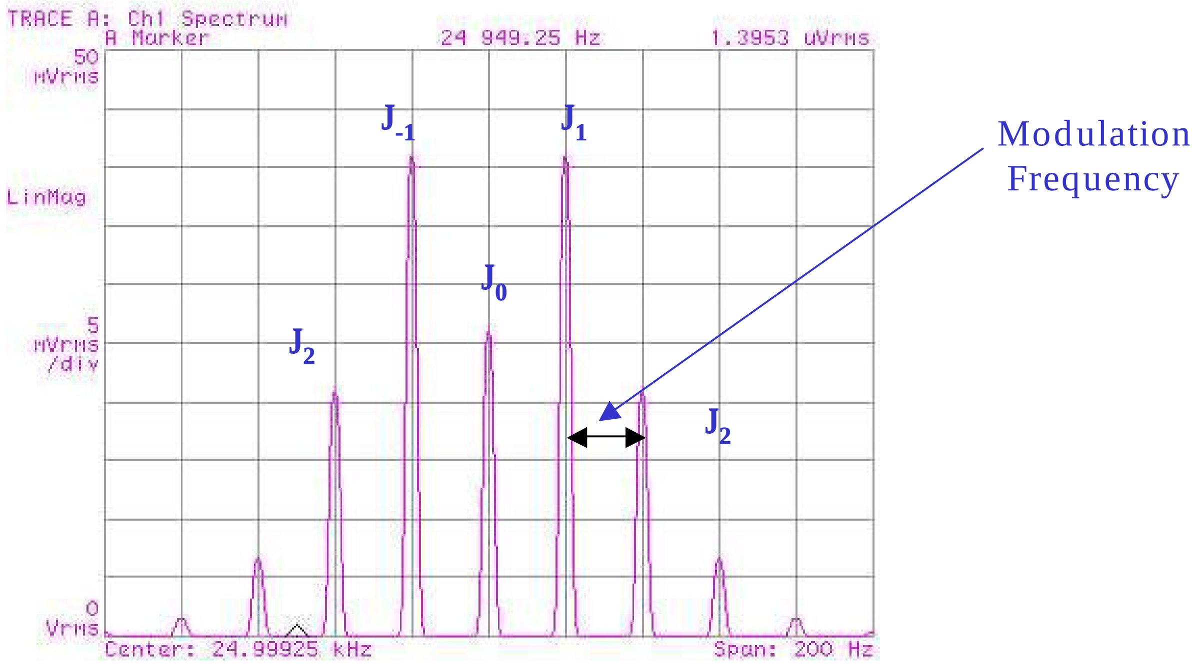
$$\omega_{k,m,n}^{\pm} = (k \pm Q_o)\omega_r + m\Omega_s + n\Omega_{mod}$$

$$Z_\pm = \left(k \pm Q_o \mp \frac{\chi}{\eta}\right) \frac{\Delta\phi_{rf}}{h} \qquad Y_\pm = \left(k \pm Q_o \mp \frac{\chi}{\eta}\right) \Delta\phi s$$

- Each synchrotron line of each betatron sideband will have a family of Bessel lines whose spacing is given by the modulation frequency and whose amplitude is given by the modulation amplitude and the chromaticity.

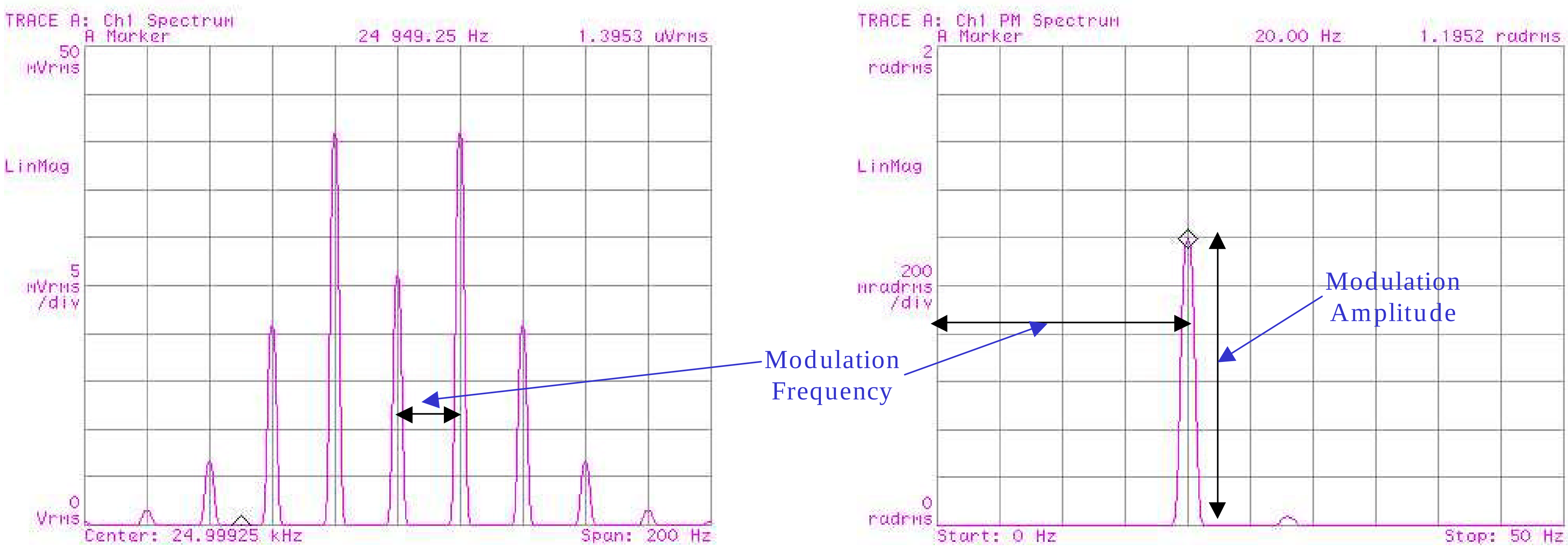
# Example Phase Modulated Spectrum

Spectrum of a phase modulated signal that has a 25 kHz carrier modulated at rate of 20 Hz with a phase modulation amplitude of 1.75 radians.




- This spectrum would show up at each of the synchrotron sidebands.
- The chromaticity could be determined by measuring the relative amplitudes of the modulation sidebands

# Demodulation of a Phase Modulated Signal

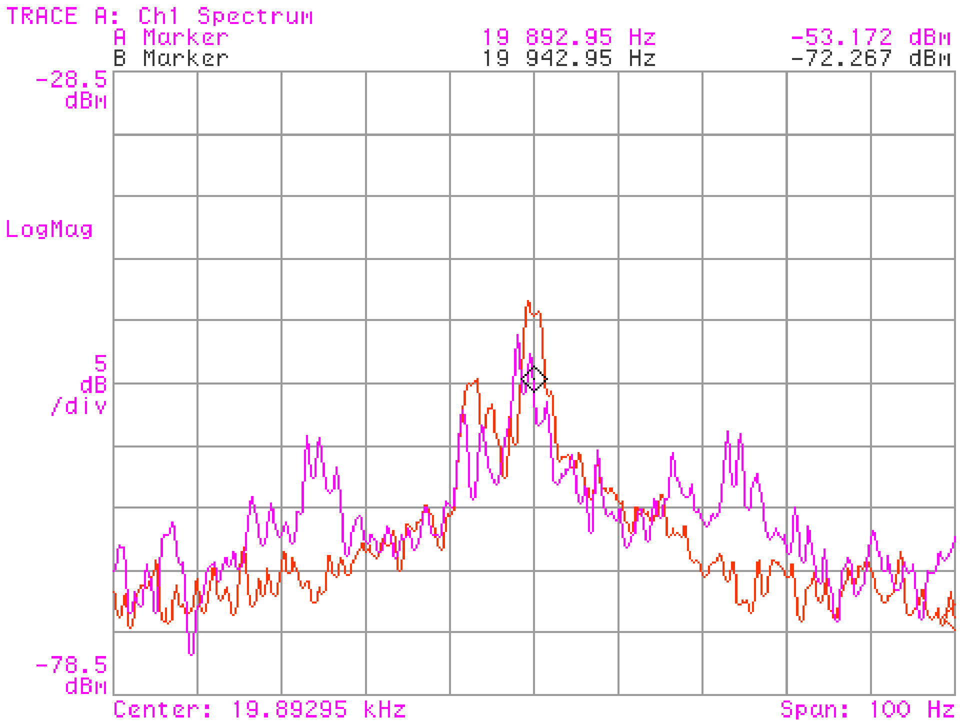



Phase Modulation Spectrum

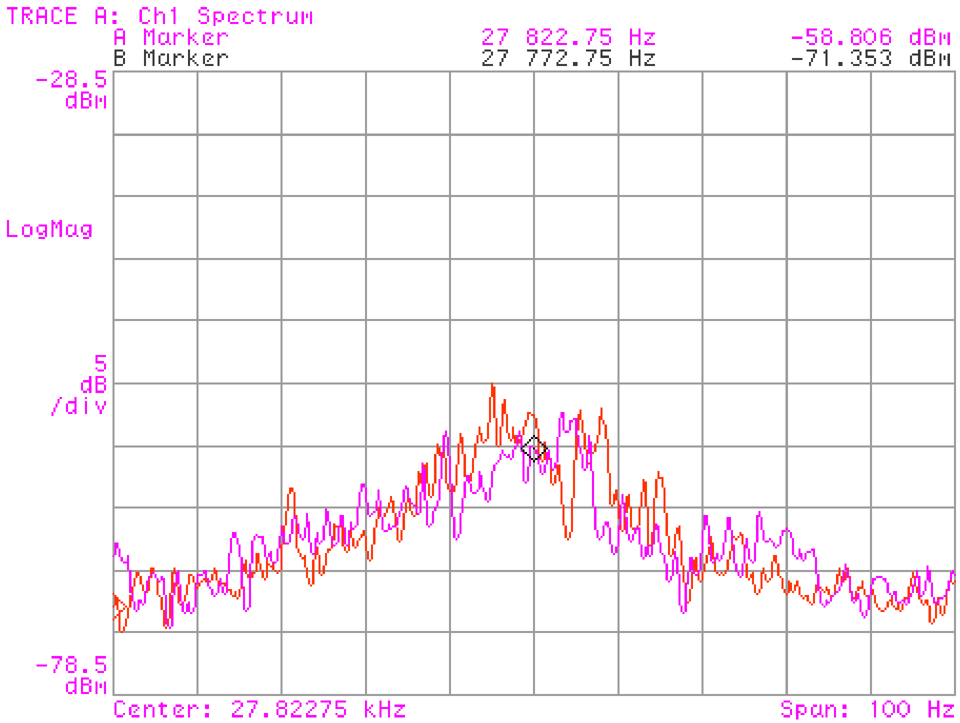
De-modulated Spectrum

- Modern vector signal analyzers can rapidly calculate and display the demodulation spectrum of a phase modulated signal.
- Note that the amplitude of the de-modulated spectrum is independent of the amplitude of the "carrier"
  - The de-modulated spectrum would be independent of the betatron and synchrotron amplitude

# TEVATRON Chromaticity Measurements using RF Phase Modulation



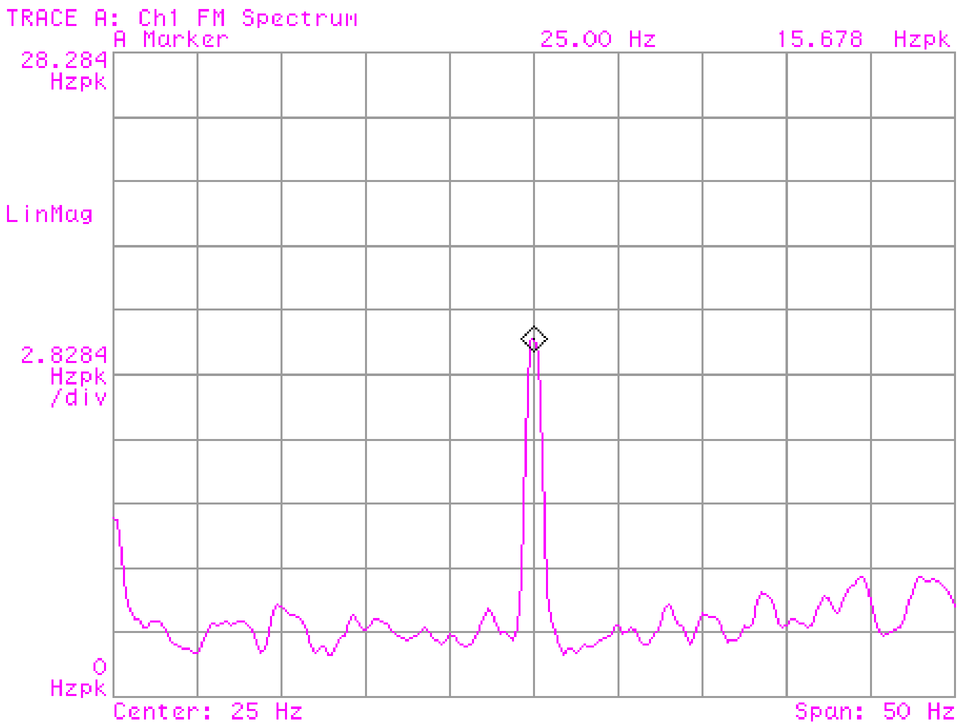

Lower Betatron Sideband



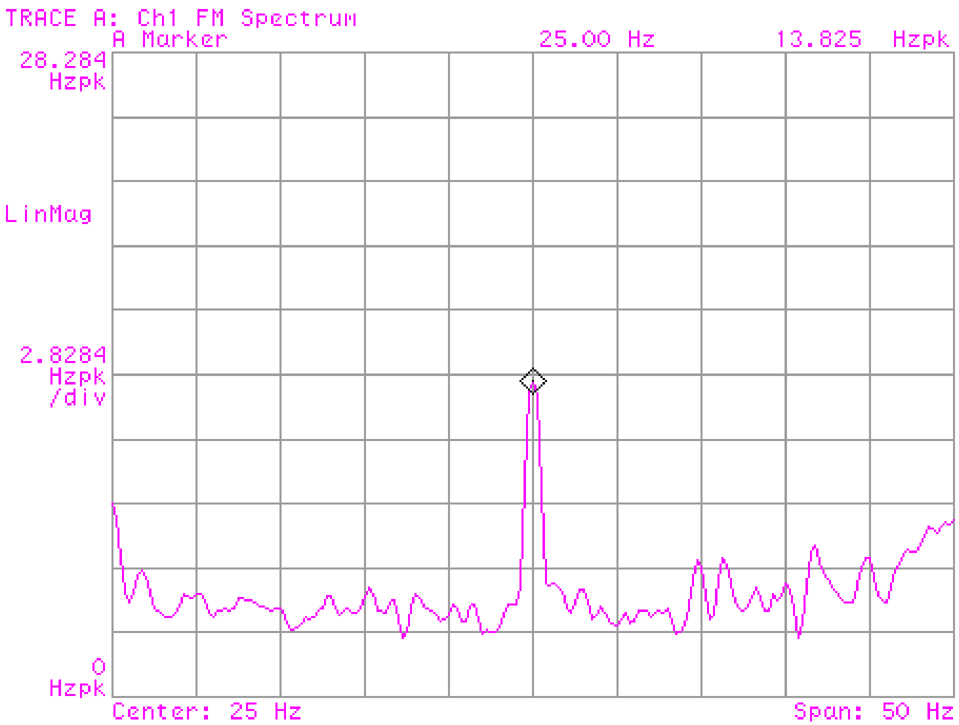

Upper Betatron Sideband

- Beam spectra of a single synchrotron line on each of the betatron sidebands
  - Red is with the RF phase modulation off.
  - Magenta is with the RF phase modulation on
    - Modulation frequency = 25 Hz
    - Modulation Amplitude = 10 degrees ($\Delta pc / pc$ = 0.003%)
- What a mess!!!!

# TEVATRON Chromaticity Measurements using RF Phase Modulation




Lower Betatron Sideband




Upper Betatron Sideband

- Demodulated spectra of the spectra on previous slide
  - Despite the messiness of the original spectra, the demodulated spectra is very clean.
  - The average amplitude can be used to determine the magnitude of the chromaticity
  - The difference of the amplitudes can be used to determine the sign of the chromaticity
- Chromaticity measurements agree quite well with traditional technique

# Summary

- The chromaticity can be accurately measured by modulating the RF phase and de-modulating the resulting betatron spectrum.
- The advantages over the present technique
  - Because the measurement is continuous, chromaticity can be measured with a much smaller (10-50x) perturbation to the beam momentum and betatron tune.
  - Technique could be easily implemented during the entire acceleration ramp without having to pause acceleration for the measurement.
- The RF phase modulation measurement technique has not been made operational in the TEVATRON and it remains to be seen whether this technique will become an operationally viable.
- Tune and chromaticity measurements might become more reliable if the beam is coherently excited with a “controlled” signal instead of relying on the “noise” of the TEVATRON to excite the betatron motion.
  - This is the basis of the TEVATRON bunch-by-bunch tune measurement system.